P.G. Semester-II Examination, 2023
MATHEMATICS

Course ID : 22154 **Course Code : MATH204C**

Course Title : Techniques of Applied Mathematics (Generalized Functions, Special Functions, Integral Equations)

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Answer any **five** of the following questions: 8×5=40

1. What do you mean by singular generalized function? Show that the Dirac delta function defined by $(\delta, \phi) = \phi(0),\ \phi \in \mathrm{D}(\mathrm{R})$ is a singular generalized function. 3+5

2. What do you mean by space of test function? Prove that

$$\lim_{\varepsilon \to 0+} \frac{1}{x + i\varepsilon} = -\pi i \delta(x) + P.\frac{1}{x}\cdot$$

(P, $\delta(x)$ and ε have their usual meaning) 3+5

[Turn Over]

3. Solve the Bessel's equation of order zero by using Frobenius method. 8

4. Define Legendre's polynomial of order n. State and prove Rodrigues' formula for Legendre's polynomial of first kind. 2+6

5. Show that

$$F(\alpha, \beta; \gamma; x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)} \int_0^1 t^{\beta-1} (1-t)^{\gamma-\beta-1} (1-xt)^{-\alpha} dt,$$

where $\alpha,\ \beta,\ \gamma$ are constants and γ is not an integer. Also show that

$$F(\alpha, \beta; \beta-\alpha+1; -1) = \frac{\Gamma(\beta-\alpha+1)\Gamma\left(\frac{\beta}{2}+1\right)}{\Gamma(\beta+1)\Gamma\left(\frac{\beta}{2}-\alpha+1\right)}.$$

4+4

6. Deduce the solution procedure of the integral equation with degenerate kernel and hence solve the equation:

$$u(x) = \sin x + \int_0^{\frac{\pi}{2}} (3\sin x \cos t + 2\cos x \sin t) u(t) dt.$$

5+3

7. Solve the integral equations:

a) $x = \int_0^x \frac{y(t)dt}{(x-t)^{\frac{1}{2}}} (3\sin x \cos t + 2\cos x \sin t) u(t)\ dt$

b) $u(t) = a\sin t + 2\int_0^t \cos(t-s)u(s)\,ds.$ 4+4

8. a) Solve the Fredholm integral equation

$$u(x) = \sin x + \int_0^{\frac{\pi}{2}} \sin x \cos y\ u(y)\,dy.$$

b) Using Laplace transformation, solve the integral equation

$$y(t) = 1 + t + \int_0^1 (t-u)\,y(u)\,du. \qquad 5+3$$

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